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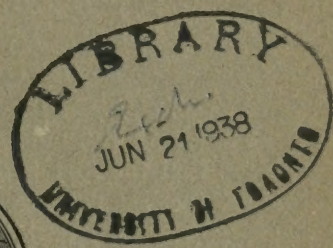
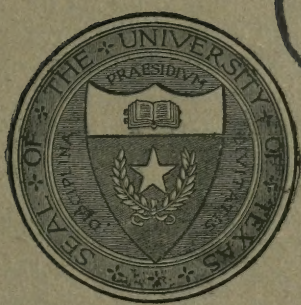
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University of Texas Bulletin

No. 1863: November 10, 1918

The Texas Mathematics Teachers' Bulletin

Volume IV, Number 1



Published by the University six times a month and entered as
second-class matter at the postoffice at
AUSTIN, TEXAS

Publications of the University of Texas

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The University publishes bulletins six times a month, so numbered that the first two digits of the number show the year of issue, the last two the position in the yearly series. (For example, No. 1701 is the first bulletin of the year 1917.) These comprise the official publications of the University, publications on humanistic and scientific subjects, bulletins prepared by the Department of Extension and by the Bureau of Municipal Research and Reference, and other bulletins of general educational interest. With the exception of special numbers, any bulletin will be sent to a citizen of Texas free on request. All communications about University publications should be addressed to the Chairman of the Publications Committee, University of Texas, Austin.

University of Texas Bulletin

No. 1863: November 10, 1918

The Texas Mathematics Teachers' Bulletin

Volume IV, Number 1

Edited by

J. W. CALHOUN,
Associate Professor of Pure Mathematics,

and

H. J. ETTLINGER,
Adjunct Professor of Applied Mathematics.

This Bulletin is open to the teachers of mathematics in Texas for the expression of their views. The editors assume no responsibility for statements of facts or opinions in articles not written by them.

CONTENTS.

Editorial	3
Course of Study and Mathematics.....J. M. Bledsoe.....	6
A Mathematical Soothsayer.....P. M. Batchelder.....	23
Orientation for Heavy Artillery.....Capt. E. J. Oglesby.....	25
Progressive Teaching of Mathematics (A Reprint)	G. W. Myers.....30
The Straight Edge.....	A. Nonymous.....41

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The benefits of education and of useful knowledge, generally diffused through a community, are essential to the preservation of a free government.

Sam Houston

Cultivated mind is the guardian genius of democracy. . . . It is the only dictator that freemen acknowledge and the only security that freemen desire.

Mirabeau B. Lamar

MATHEMATICS AT THE T. S. T. A.

To the Mathematics Teachers of Texas:

Like all things pertaining to human activities and affecting organized society, mathematics has been profoundly influenced by the great war. This effect has been felt in many ways. One has been the withdrawing of many of the best men and women from the ranks of the teachers of mathematics, the men to serve in the Ordnance Department of the Government, the women to work in the Bureau of Standards and allied enterprises of the departments at Washington. This has had the collateral effect of making the subject of mathematics and its teaching a matter of greater public interest and at the same time reducing the average efficiency of mathematics teachers in general.

In great crises like the present one, things of a non-essential nature get scant consideration. Subjects whose presence in curricula is due to fad, fashion, four-flushing, or favoritism are withered at the first hot blast of war. Nothing that cannot give an account of itself that is positive, immediate and indubitable has any chance for consideration.

Among the things that, from the very beginning, have been counted on to help win the war is mathematics. Everybody who knew anything of the subject and its applications knew that it was destined to play an important part. Mathematics is consulted by the maker and by the user of big guns. It gives invaluable aid to the engineer who lays out defenses. Without it navigation is guesswork. Mapmaking depends almost wholly on its aid. The aviator learns the distance and the speed of his foe by the principles of elementary plane geometry. On the occasion of the sinking of the first submarine by a gun mounted on the deck of a U. S. steamer, the comment of the captain, quoted in the newspapers of the country the following morning, was "IT WAS A FEAT OF PURE MATHEMATICS."

It is going to be a long time, in the judgment of the writer, before any alleged educator arises and questions the validity of the claim of mathematics to a prominent place in any educational program. This does not mean that neither the content nor the method of teaching will be subject to any change. There

is little doubt that both may undergo some modification. But change and modification are signs of life and are not characteristic of fossilization. It is one of the duties of those of us who love the subject and who believe in it to be alert to find in what way and to what extent change is desirable and improvement possible.

One of the best ways that we can accomplish this result is by conference and discussion. There comes only one opportunity each year for such a coming together and taking counsel with one another. This is the meetings of the Mathematics Section of the Texas State Teachers' Association. This meeting will be held the present year at Dallas. It will embrace the days of November 28-30. The Mathematics Section will have two meetings, probably Thursday and Friday.

The programs are not as yet completely made but enough progress has been made to give a clear indication of the nature and the excellence of the discussions to be had at that time. Most of the papers will be presented by teachers actually engaged in high school work in mathematics. The papers will be short and will be numerous. They will touch many subjects. They will be presented out of the experience of successful teachers. They will set forth what the teachers have gathered in their work and not what they have read about in a book written by some doctrinaire. Any person presuming to occupy more than eight minutes will be classed as *irrational* or *(ab)surd*. One attempting to take the time of the assembly to apologize will be promptly *eliminated*. All speeches will be reduced to their *lowest terms*. No *extraneous* solutions of any questions will be allowed.

We are going to have a meeting *numerically* and *absolutely* large, *plus* plenty of "pep" and enthusiasm, *minus* slackers and quitters, *divided* into two complete programs, *multiplied* by all the *factors* that go to make a successful teacher and a clear thinker, and *raised to the n th power*.

Any teacher of mathematics who misses these meetings is going to suffer a loss. Everyone who comes is going to get an elevation of spirit (doubtless accompanied by a raise in salary) that will stay with him through the remaining months of the school year at the very least. It will be more than worth the

time and money it will cost to be at this meeting and to meet the best teachers in your field who are going to gather there.

A few of the people who will be on the program are: Mr. W. O. DeWees, Weathford High School; Mr. C. O. Calloway, Waco High School; Miss Mattie Watson, Mexia High School; Mr. E. L. Dohoney, Jr., Paris High School; Mrs. Helen Bolton, Brenham High School; Miss Goldie P. Horton, University of Texas; and Mr. J. B. Gay, State School for the Blind. But these are by no means all.

A few of the subjects to be discussed are: Teaching Geometry to the Blind, Can I Interest All My Pupils in Geometry? Mathematics and The Great War, Girls vs. Algebra, Talking Over the Heads of Beginners, Factoring in First Year Algebra, The Value of Practical Problems, Number Systems and Numerals, and a lot more just as good.

If you do not know the people named, it will be worth your while to come and meet them. If you do know them, you will need no urging to come. There is not a subject on this list that is not of interest to every teacher of mathematics, and there is not a subject named that any high school teacher of mathematics cannot understand.

Let us all get together. If you are one of the faithful who always attend, come this time. If you sometimes go, let this be one of your times. If you have never been in your life—well, it's never too late to mend. Come, and see what you have been missing.

Remember the place—Dallas.

Remember the time—November 28-30.

Cordially yours,

J. W. CALHOUN,

Chairman Mathematics Section T. S. T. A.

“SOME OBSERVATIONS ON COURSE OF STUDY, AND MATHEMATICS IN PARTICULAR”

PREPARED BY J. M. BLEDSOE, OF THE DEPARTMENT OF MATHEMATICS,
EAST TEXAS STATE NORMAL COLLEGE, COMMERCE, TEXAS

“PURPOSES OF A TEACHER-TRAINING SCHOOL”

1. To train teachers for the public schools, both elementary and secondary.
2. To educate and prepare men and women for the great profession of teaching, a work second to none among all the vocations.
3. To develop methods and to serve as model training schools for young men and women who contemplate the work of teaching.

“ESSENTIAL CHARACTERISTICS OF A PROGRESSIVE AND SUCCESSFUL TEACHER”

1. He must be thoroughly educated in the subject matter of the department in which he wishes to teach.
2. He must be able to understand the abilities, habits, inclinations, desires, and aspirations of the pupils he wishes to teach, and be able to present joyfully and effectively the instruction to these pupils.
3. He must be familiar with and able to make effective use of the very best methods of instruction.

The Teacher-Training School then, is not only to train the teacher from the professional point of view, but to instruct him from the academic point of view.

“POINTS RELATIVE TO COURSE OF STUDY AND MATHEMATICAL TRAINING”

No man may be truly classed as an educated man who does not know something about a great many things, and a great many things about something. In other words, a man cannot be

called truly educated who does not know that such a character as Alexander the Great existed, or that the Battle of Marathon was fought; such facts are learned from the study of history, therefore, it is necessary to know something about history in order to be educated. Likewise, a man who is unfamiliar with the location of the Amazon River valley, the correct structure and use of the sentence, the simple application and use of the lever, and a proper conception of sizes and measurements of ordinary magnitudes could not properly be called an educated man.

Therefore, for a man to be educated he must be familiar with at least the common every-day facts relative to a great many subjects, and must know a great many things about at least one subject.

The time to specialize does not come early in life to most students; therefore, it is the duty of the Course of Study Committee to arrange the subject matter so as to give each student the advantage of the broadest training possible, in order that a wide vision of the field of knowledge may be obtained.

The conceptions of education for ages has been that it broadens and develops the power of the mind. The maxim of the ancients was: "Reason expands and elevates the Mind." The generally accepted idea of education is that it is that process or acquirement which enables the individual to more readily originate and apply the principles of harmony and force to the solution of life's problems. We think of the educated mind as one strong and capable of doing things effectively, which the uneducated mind is unable to observe or accomplish.

The question of "knowledge of most worth" is one that has not been definitely settled. This matter may be said to depend on the vocation of the individual. A man who expects to follow the profession of law-making will naturally feel that a knowledge of civics and jurisprudence will be of more value to him than the details of chemistry or mineralogy. The question of relative value of subjects in the school curriculum has been discussed for many centuries and by many educators. Herbert Spencer wrote an extensive thesis on the subject of "Knowledge of Most Worth," but did not arrive at any very satisfactory conclusion even to himself; however, stated that the natural sciences were

possibly of greater value than any other department of knowledge.

All knowledge is useful when the circumstances of life call for its application. Since we never know just when any particular fact will be called for, it is the part of wisdom to lay a broad foundation in order that we may not be disappointed in our ability to originate and successfully solve the problems of life as they come.

The study of mathematics furnishes not only suitable materials for developing the powers of the mind, but gives a system of knowledge which is both interesting and serviceable in its application.

Mathematics is the mother of all the sciences, yet it is true in some cases that she must offer an apology for her appearance on the course of study and in the class room.

Mathematics must be studied early in life, generally speaking, in order that any high degree of attainment may be reached. This is not so necessary in other subjects. A man may wait till late in life and master other subjects, but it has been found that the individual who starts late in life, finds it very difficult, if not impossible, to reach any degree of distinction in the field of mathematics.

Courses in mathematics must come in consecutive and logical order, and not too far apart in time, if the greatest advantage is to be derived. In other words, it would be next to impossible for a student to begin the subject of Trigonometry in the first year of high school; yet a student may pursue satisfactorily a course in third-year college history in the first year.

The Herbartian theory supposed that there is no transfer of power from the study of one subject to that of another, and the pedagogical pendulum swung to the extreme against the theory of formal discipline. Thorndike and Judd, two of the leading modern authorities on educational questions dispute on this point of "formal discipline." Judd says: "The problem of education thus turns out to be the problem of generalizing experiences." "The generalizing of experience is a qualitatively new fact wherever it appears."

"A subject which gets itself so organized that it rotates around its own center immediately becomes formal." "The cultivation of this power of generalization is the most important achievement in the student's education." Let any one attempt to find a subject in the whole curriculum to which Judd's ideas of generalized training could be more easily applied than in the field of mathematics.

It is doubtful whether any other educational item has received more careful attention in the last fifteen years than the teaching of mathematics. In 1912 the United States Bureau of Education published a special bulletin containing a bibliography of 1,849 titles, all pertaining to books, memoirs, and pamphlets written since 1900, either here or abroad, on the teaching of mathematics. An International Commission was created to unify and standardize the mathematical teaching of all the civilized countries. Since the year 1897 there have been held six International Congresses, in which the membership has increased from 242 to over 600; and the number of countries participating grew from 16 to 30.

Mathematics is the basis of practically our entire material civilization. Without it we should return to primitive conditions to an extent which is almost impossible to exaggerate. Our entire business life, including manufacturing and transportation, our engineering enterprises, architecture, the sciences such as physics, chemistry, economics, astronomy, statistics, and even psychology, are absolutely dependent on the processes of mathematics.

In recent times there has developed a rapidly growing interest in the natural sciences. The old method of the Greeks to look inward for knowledge has been largely replaced by the method of looking outward and depending upon patient experimentation for the foundation of knowledge. At the same time mathematics has been largely transformed from a non-empirical science to a semi-experimentation science. Many excellent works have been written by such modern educators as Cajori, Lobacevsky, Slaughter David, Eugene Smith, and others which show the fallacy of the old institutional methods of teaching mathematics, and the wisdom of the more rational method of experimental

demonstration. Had Lord Macauley, when at school, been supplied with mathematical text books which showed the relation of the subject in question to real life, he would never have written home in despair, "Oh for words to express my abomination of that science," nor later had so much reason to regret the want of a "knowledge of physics and mathematics."

Mathematics has just kept in touch with the great scientific activities of the age. Just as it rendered most useful service to the ancient Greeks in their philosophical speculations, so it now stands at the side of the natural scientist to render indispensable assistance in all his experiments and the application of his principles. In fact, the relation between mathematics and the natural sciences is so close, that there is a tendency in some of these sciences to adopt the saying of Plato, slightly changed, as a part of their entrance requirements, which stated in present-day needs is "Let no one ignorant of mathematics come under my roof." And there seems to be no good reason to suppose that the usefulness of mathematics to the natural sciences will ever wane. On the other hand, this usefulness may reasonably be expected to increase with the growth of knowledge.

As an evidence of the development of the idea of recognition of the value and importance of mathematics in the school curriculum, one is only required to note the history of mathematics in some of our leading institutions. "In 1802 the standard for admission to Harvard College was raised. In mathematics a knowledge of arithmetic to the 'rule of three' was required."

Thus, in 1803, for the first time had it become necessary, according to regulations, for a boy to know something in arithmetic, however, must have been very meager, for it has been found from the diary of a student of the Freshman class of 1807 that arithmetic continued to be taught during the first year at college. After 1816 the whole of arithmetic was required for admission. From this small beginning in our course of study a little more than a century ago, the requirements in mathematics for entrance into our colleges have advanced from practically nothing until now the admission requirements include not only the whole of arithmetic (which presupposes the work of 5 or 6 years in the grades and in the high school), but at least $1\frac{1}{2}$ year's work

in algebra and one year in plane geometry. It is significant to note that many of our leading colleges and universities have, in the last few years, raised the entrance requirement in mathematics; the University of Texas increased its mathematics requirements for the present year one-half unit.

A proper training for the teaching of mathematics in our secondary schools requires a wide range of subject matter as well as a thorough professional training and successful observation and experience. The subject of mathematics is considered by leading educators as possibly the most difficult subject to teach properly. And a wide range of preparation and professional skill is absolutely necessary to successful teaching of the subject.

The Committee of Ten and the Committee on College Entrance requirements in mathematics in leading colleges and universities have been published; and these committees recommend that a Standard College Course in mathematics should "Include (besides thorough courses in the subjects of secondary school instruction themselves, arithmetic, plane and solid geometry, algebra, the elements of trigonometry), good courses in trigonometry (surveying desirable), college algebra, plane analytic geometry, the differential and integral calculus, the theory of equations, including determinants; courses in the history of mathematics, and the elements of analytic mechanics; some work in theoretic and practical physics should be included.

(Resolution III of the Committee of College Entrance Requirements is as follows: "Resolved, That the teachers in the secondary schools should be college graduates or have the equivalent of college education." Report, p. 30.)

The teachers trained in the State schools for the teaching in the High Schools should be thoroughly prepared in the subject matter which is taught in these schools; and no teacher is thoroughly prepared to teach in the high school if he has had no training beyond the mere subjects which are to be taught. In addition to an extensive training in subject matter two or three years beyond what is given in the high school the teacher should also have ample professional training.

What do the high schools of Texas give to be taught in their curricula? See page on Texas High Schools, p. 8.

What do the Texas Colleges and Universities consider the minimum requirements for a first class course in mathematics?

What do the leading Colleges and Universities of America give in the subject of mathematics?

What is given as the average amount of mathematics by the best State Normal Schools of other States? See pages 11-14.

“MATHEMATICS OFFERED IN TEXAS HIGH SCHOOLS”

Dallas

Required for graduation: 16 high school units.

Prescribed 4—English, 3; Mathematics, 1; Elective, 12.

Mathematics Offered: Algebra, $1\frac{1}{2}$; Commercial Arithmetic, $\frac{1}{2}$; Plane Geometry, 1; Solid Geometry, $\frac{1}{2}$; Trigonometry, $\frac{1}{2}$; Book-keeping, 1; Total, 5.

Waco

Required for Garduation: 16 high school units.

Prescribed 9—English, 4; Mathematics, $2\frac{1}{2}$; History, 1; Science, 1; Political Science, $\frac{1}{3}$; Elective, 7.

Mathematics Offered: Algebra, $1\frac{1}{2}$; Plane Geometry, 1; Advanced Algebra, $\frac{1}{2}$; Solid Geometry, $\frac{1}{2}$; Trigonometry, $\frac{1}{2}$; Arithmetic, $\frac{1}{2}$; Book-keeping, $1\frac{1}{2}$; Total, 6.

Corsicana

Required for Graduation: 16 high school units.

Prescribed 7—English, 4; Mathematics, 3; Elective, 9.

Mathematics Offered: Algebra, 2; Plane Geometry, 1; Advanced Arithmetic, $\frac{1}{2}$; Solid Geometry, $\frac{1}{2}$; Trigonometry, $\frac{1}{2}$; Total, $4\frac{1}{2}$.

Bonham

Required for Graduation: 19 high school units.

Prescribed 14—English, 4; Mathematics, 3; History, 3; Latin, 4; Elective, 5.

Mathematics Offered: Algebra, 2; Plane Geometry, 1; Solid Geometry, $\frac{1}{2}$; Advanced Arithmetic, $\frac{1}{2}$.

Required for Graduation: 19 high school units.

Prescribed 12—English, 4; Mathematics, 4; History, 4; Elective, 7.

Mathematics Offered and Required:

First Year: Algebra 5 periods per week for whole year.

Second Year: Algebra First Term; Plane Geometry; Second Term, 4 periods.

Third Year: Plane Geometry First Term; Solid Geometry Second Term; 4 periods.

Fourth Year: Algebra First Term; Trigonometry Second Term; 4 periods.

Commerce

Required for Graduation: 16 high school units.

Prescribed 12—English, 4; Mathematics, 3; History, 3; Science, 2; Elective, 4.

Mathematics Offered: Algebra, 2; Plane Geometry, 1; Advanced Arithmetic, $\frac{1}{2}$; Solid Geometry, $\frac{1}{2}$; Total, 4.

“MATHEMATICS REQUIRED FOR BACHELOR'S DEGREE IN TEXAS COLLEGES”

Southwestern University: 3 Majors of college work (120 recitation hours) for any degree—Sol. Geom., Trig., Col. Alg.

Simmons College: 3 Courses (108 recitation hours) required in all groups. 4 Courses (144 recitation hours) required in Sci. group.

Trinity College: Course 1—Solid Geometry; Course 2—Algebra; Course 3—Algebra; Course 4—Advanced Algebra; Course 5—Plane Trigonometry.

Baylor Female College: 2 Courses (120 rec. hrs.) required for any degree.

Baylor University: Minimum of 2 Majors (120 rec. hrs.) in all groups; 3 Majors required in science group (180 rec. hrs.)

Howard Payne College: Minimum of $1\frac{2}{3}$ Courses (180 hrs.) in all groups; 2 Courses (216 hrs.) in scientific group.

Southern Methodist University: Students presenting minimum entrance requirements of Algebra $1\frac{1}{2}$, Plane Geometry 1; are required to take 3 courses—Solid Geometry, Trigonometry, and Introductory Analytical Geometry—108 recitation hours. Students may receive credit for Solid Geometry or Trigonometry, or both on entrance, and only required to take one course in either Advanced Algebra or Anal. Geom.

Austin College:

Required: First Year, 4 hrs. per wk. (144 hrs.), Solid Geom. Trig.; Second Year, 3 hrs. per wk. (108 hrs.), Adv. Alg. Surveying.

Elective: Third Year, 3 hrs. per wk. (108 hrs.), Analytical Geom.; Fourth Year, 3 hrs. per wk. (108 hrs.), Dif. and Integral Cal.

Rice Institute: 3 Terms (108 hrs.), Trig., Adv. Alg., Anal., required of all.

Texas Christian University: 3 Terms (108 hrs.), required for all bachelor' degree.

Survey of 76 Colleges and Universities. Bulletin 1911, No. 7.

1. Very few students are allowed to graduate without a year's mathematics in college, ordinarily at least solid geometry and trigonometry.

2. Out of the 76 Colleges and Universities, only two do not prescribe any college work in mathematics—and only does not offer as electives a number of college courses in mathematics for the bachelors degree, p. 15.

“FACTS RELATIVE ENTRANCE REQUIREMENTS”

I. The University of Texas— $14\frac{1}{2}$ units required. (A type of State University and Co-educational Institution.)

1. Prescribed 10—English 3; History 2; Foreign Language 2; Mathematics 3.
2. Elective $4\frac{1}{2}$ — $1\frac{1}{3}$ of which may be mathematics.

- II. Yale University— $15\frac{1}{2}$ units required. (A type for men only).
1. Prescribed $11\frac{1}{2}$ —English 3; Latin 4; Modern Language 2; Mathematics $2\frac{1}{2}$.
 2. Elective 4—1 of which may be mathematics.
- III. Wellesley—15 units required. (A type for women only).
1. Prescribed 14—English 3; History 1; Latin 4; Mathematics 3; Greek or a Modern Language 3.
 2. Elective 1—which may be mathematics.
- IV. Boston Technical Institute—14 units required. (A type of the high class technical school).
1. Prescribed 12—Mathematics 3; Physics 1; History 1; French 2; German 2; English 3.
 2. Elective 2—1 of which may be mathematics.
- V. Leland Stanford Jr.—15 units required. (A type of Western freedom).
1. Prescribed 3—English 3.
- Elective 12—5 of which may be mathematics.

“GRADUATION REQUIREMENTS—LEADING INSTITUTIONS”

The University of Texas

Admission Units— $14\frac{1}{2}$.

1. Prescribed 10—English 3; History (or History and Civics) 2; Mathematics 3; Foreign Language 2.
2. Elective $4\frac{1}{2}$ — $1\frac{1}{2}$ of which may be mathematics.

For Graduation 20 Units Required.

1. Prescribed $8\frac{1}{3}$ —English 2; Mathematics 1; Foreign Language 2; Natural Science 2; Economics or Government 1; Philosophy or Psychology $\frac{1}{3}$; (Must complete 5 advanced courses).
2. Elective $11\frac{2}{3}$ —4 of which may be mathematics.

The University of Missouri

Admission Units Required—15.

1. Prescribed 6—English 3; Mathematics 1; Foreign Language 2.
2. Elective 9—3 of which may be mathematics.

For Graduation—Total of 124 hours required.

1. First Two Years: (a) 6 hours of English; (b) 5 hours of history; (c) 5 hours of ancient languages and literature; (d) 5 hours of modern languages; (e) 3 hours of mathematics; (f) 5 hours of physical science; (g) 5 hours of biological science; (h) elective 28 hrs.
2. Last Two Years: (a) One Major of at least 24 hours; (b) one minor of at least 12 hours; (c) electives 26 hours.

The A. and M. College of Texas

Admission Units Required: 14.

1. Prescribed $5\frac{1}{2}$ —Mathematics $2\frac{1}{2}$; English 3.
2. Elective $8\frac{1}{2}$ — $11\frac{1}{2}$ of which may be mathematics.

For Graduation—Engineering Department.

1. First Two Years: Solid Geometry; Advanced Algebra; Trigonometry; Analytical Geometry; Calculus, Dif. and Integral.
2. Last Two Years: Applied mathematics during whole time.

“LEADING STATE NORMALS—OTHER STATES”

Southern Illinois State Normal University

Carbondale, Illinois

Required for Bachelor's Degree—24 units. (One unit means one semester's work of 5 one-hour periods per week).

1. Prescribed 12 units—Mathematics 3; History 3; High School Education 1; High School Practice 1; Psychology 1; English 1; Biological or Physical Science 2.
2. Elective 12 units—Latin 6; Modern 6; Mathematics 3; Biological or Physical Science 4; English 2; Geology 1; Astronomy 1; Art 3.

Warrensburg, Missouri, State Normal

Entrance Unites Required: 16.

Prescribed 10—English 3; Mathematics 2; History 2; Science 2;
Technical Subjects 1.

Elective 6—English 1; Mathematics 2; History 2; Science 2;
Foreign Language 4; Technical Subjects 2; Education
1½.

For Graduation—B. S. Degree in Education. Requirements.

Requirements for Graduation:

Education	27½ hrs.
English	T hrs.
A Major.....	25 hrs.
A Minor.....	10 hrs.
Electives	52½ hrs.
Total	120 hrs.

Hays, Kansas, State Normal School

For B. S. degree in Education.

One hour equals 4-55 min. periods per week for 5 weeks.

One unit equals 5-45 min. periods per week for one year.

1. Mathematics required; 8 hours credit.

2. Mathematics Offered:

(1) Preparatory: 2½ units.

(2) College: 24 hours.

3. Content of Work Offered:

(1) Preparatory: (a) General Arithmetic ½;
Business Arithmetic ½; Academic 1½;
Total 2½.

(2) College; Plane Geometry 3 hours; Ad-
vanced Algebra 6 hours; Plane and Spheri-
cal Trigonometry 3 hours; Farm Survey-
ing 4 hours; Pedagogy of Arithmetic 2
hours; Pedagogy of High School Mathe-
matics 2 hours; Surveying 4 hours; Analytic
Geometry 2 hours; Total of College Math-
ematics Offered 26 hours.

Indiana State Normal School

Terra Haute, Indiana

For the A. B. degree in Education.

One course equals 4-55 min. periods per week for 12 weeks.

1. Mathematics Required: (a) Preparatory 9; (b) College 3.
2. Mathematics Offered: (a) Preparatory 9; (b) College 14 courses.
3. Content of Work Offered:
 - (1) Preparatory: Arithmetic 3; Academic Algebra 3; Plane Geometry 2; Solid Geometry 1; Total 9.
 - (2) College: College Algebra 2; Trigonometry 1; Analytic Geometry 2; Calculus 2; Industrial Mathematics 1; Differential Equations 1; Theory of Equations 1; Analytic Mechanics 1; Theory of Groups 1; Projective Geometry 1; History and Teaching of Mathematics 1; Total of College Courses Offered 14.

For students who wish to teach mathematics in the smaller schools: College Algebra, Trigonometry and Methods in Geometry.

For students who desire to become special teachers of mathematics: In addition to the subjects named above, Analytic Geometry, Theory of Equations, Advanced Arithmetic, Calculus (Differential and Integral), Surveying.

Eastern Illinois State Normal School

Charleston, Illinois

This school gives a four-year course, or two college years' work.

Mathematics Required: Algebra 3 terms 5 hrs. per week; Plane Geometry 2 terms 5 hrs. per week.

Mathematics Offered as Elective: Solid Geometry 1 term of 4 hrs. per week; Plane Trigonometry 1 term of 4 hrs. per week; Analytic Geometry 1 term of 4 hrs. per week.

Missouri State Normal School

Cape Girardeau, Missouri

Mathematics Required: Algebra (3 recitations per week); Plane Geometry 2 terms; Solid Geometry 1 term; Methods in Advanced Arithmetic 1.

Mathematics Offered as Elective: College Algebra 2 terms; Trigonometry 1 term; General Astronomy 2 terms; Analytical Geometry 3 terms; Calculus 3 terms; Theory of Equation 1 term.

South Georgia State Normal College

Valdosta, Georgia

(This school is for girls only)

Offers following two courses:

I. Normal Course.

First Year: Plane Geometry—1 unit, 4 periods per week for year.

Second Year: Applied Mathematics— $\frac{1}{2}$ unit.

Fourth Year: Arithmetic and Methods $\frac{1}{2}$ unit.

II. Non-Professional Course.

First Year: Plane Geometry—1 unit.

Second Year: Advanced Algebra $\frac{1}{2}$; Solid Geometry $\frac{1}{2}$ unit.

Third Year: Plane Trigonometry $\frac{1}{2}$.

The Normal Course offers no mathematics of College rank, while the Non-Professional Course gives 216 recitation hours to the three courses included in a standard year, viz., Advanced Algebra, Solid Geometry, and Trigonometry.

Iowa State Teachers College

Cedar Falls, Iowa

Admission Units Required: 15.

For the B. A. Degree in Education:

Mathematics Required: Solid Geometry 2 hours; College Algebra 5 hours; Trigonometry 5 hours.

Mathematics Offered in Addition to Above: College Algebra 3 hours; Trigonometry 2 hours; Surveying 3 hours; Theoretical Mechanics 3 hours; Teaching of Secondary Mathematics 3 hours; History of Mathematics 2; Analytical Geometry 5 hours; Calculus I, 5 hours; Calculus II, 3 hours; Modern Geometry 3 hours; Theory of Number 2 hours; Differential Equations 3 hours.

Kansas State Normal School

Emporia, Kansas

Admission Units Required: 15.

Mathematics Required for the Degree of B. S. in Education: 3 hours Col. Math.

Mathematics Offered: Advanced Arithmetic 4 hours; Teaching of Arithmetic 2; Teaching of Commercial Arithmetic 2 hours; Advanced Algebra and its Applications 3 hours; Solid Geometry 3 hours; Trigonometry 3 hours; Surveying 3 hours; College Algebra 5 hours; College Algebra 3 hours; Analytics I, 4 hours; Analytics II, 3 hours; Calculus II, 3 hours; Calculus II, 4 hours; Calculus III, 3 hours; Teaching of Secondary Mathematics, 4 hours; Astronomy 2 hours; History of Mathematics 2 hours; Solid Analytical Geometry 2 hours; Differential Equation 3 hours; Higher Algebra 2 hours. Students may major in Mathematics as in any other Department.

Colorado State Teachers College

Greely, Colorado

Admission Units Required: 15.

This school grants the B. A. degree in Education on completion of the four-year course, and the M. A. degree in Education after the completion of one year of graduate work.

Mathematics Required: One year of College Mathematics, or may select the Major subject as Mathematics, in which a minimum of 48 hours or a maximum must be completed in the Senior College.

Mathematics Offered: Solid Geometry 4 hours; Plane Trigonometry 4 hours; Trigonometry 2 hours; Surveying 2 hours; College Algebra I, 2 hours; College Algebra II, 4 hours; Analytical Geometry 5 hours; Teaching of Arithmetic 2 hours; Teaching of Arithmetic II, 4 hours; Country School Arithmetic 4 hours.

"SUMMARY OF FINDINGS ON MATHEMATICS CURRICULA"

Mathematics in Texas Public Schools

Required for Graduation: (A) Mental and Practical Arithmetic; (b) Higher Arithmetic; (c) Advanced Arithmetic or Commercial Arithmetic; (d) Elementary and High School Algebra; (e) Plane Geometry; (f) Solid Geometry or Plane Trigonometry.

Units Offered as Electives: (a) Advanced Arithmetic; (b) Commercial Arithmetic; (c) Bookkeeping; (d) Advanced Algebra; (e) Solid Geometry; (f) Plane Trigonometry.

"Entrance Requirements in State Normals and First Class Colleges"

Minimum Requirements: (a) Algebra $1\frac{1}{2}$; (b) Plane Geometry 1; (Presuppose the completion of Practical and Higher Arithmetic).

Elective Entrance Units: (a) Advanced Algebra; (b) Bookkeeping; (c) Solid Geometry; (d) Plane Trigonometry.

“Mathematics Curricula in State Normals and First Class Colleges”

Required for Graduation, Bachelor's Degree:

Minimum for Any Degree—108 recitation hours of College work above entrance requirements.

Maximum for Students Majoring in Science or Mathematics—From 324 to 648 recitation hours of college work above entrance requirements.

Content of College Courses Offered:

Technical: (a) College Algebra; (b) Advanced Arithmetic; (c) Advanced Plane Geometry; (d) Bookkeeping; (e) Solid Geometry; (f) Plane Trigonometry; (g) Industrial Mathematics; (h) Plane Surveying; (i) Farm Surveying; (j) Elementary Mechanics; (k) Strength of Materials; (l) Industrial and Insurance Statistics; (m) Descriptive and Mathematical Astronomy; (n) Plane and Solid Analytical Geometry; (o) Differential Equations; (q) Theory of Equations; (r) Theory of Groups; (s) Projective Geometry—25 of our units.

Professional: (a) Methods in Arithmetic; (b) The Teaching of Elementary Mathematics; (c) The Teaching of Secondary Mathematics; (d) The Pedagogy of Farm Mathematics; (e) The History of Mathematics—5 of our units. Total 30 units.

A MATHEMATICAL SOOTHSAYER

BY P. M. BATCHELDER

Last August there appeared in the newspapers a mathematical "proof" that the present war will end during the year 1918, based on the date of birth, first year of leadership, present age, and length of holding office of eight leaders of the principal Allies. It was contained in a letter sent to an officer at the Great Lakes Naval Training Station in Illinois.

The argument of the prophet (who for reasons best known to himself prefers to remain anonymous) is as follows: he adds together the four numbers representing the date of birth of the leader, the date of his accession to power, the number of years he has ruled, and his age, and finds that the sum, when divided by 2, is 1918 in each case. Thus for President Wilson:

Born	1856
Elected President	1912
Years ruling	6
Age	62
	3836

The sum 3836, when divided by 2, gives 1918. Similarly for King George of England:

Born	1865
Came to throne.....	1910
Years ruling	8
Age	53
	3836

which likewise gives 1918 on division by 2. The same result is obtained for the President of France, the Kings of Italy, Belgium, and Serbia, the Czar of Russia, and the Emperor of Japan.

Such a remarkable coincidence as this evidently has momentous implications, although the exact logical processes by which our prophet connects it with the end of the war remain enshrouded in mystery. Indeed, we must marvel at his modesty in naming only eight instances; for a little investigation discloses the fact the same procedure may be applied with identical

results not only to the rulers named above, but also to all the statesmen of the warring countries and to all the generals commanding their armies. Indeed, it holds good for every person who holds an office or position of any kind whatever, from the Ameer of Afghanistan to the Assistant Secretary to the Mayor of Podunk. Thus we are fairly overwhelmed with the amount of evidence.

Moreover, the mathematics involved in the argument, when submitted to the scrutiny of a professional mathematician, is discovered to admit of simplification. For definiteness let us consider the case of President Wilson. He was born in 1856 and is 62 years old, and

$$1856+62=1918;$$

also, he was elected President in 1912 and has been in office 6 years, and

$$1912+6=1918.$$

Thus we obtain the desired result without finding it necessary to add more than two numbers at a time or to make any use at all of the process of division. Incidentally we have strengthened the argument by obtaining 1918 in two ways instead of one. The reader will easily verify that any other case admits of the same treatment.

Another striking feature to which the prophet fails to call attention is the perennial validity of his argument. Suppose that by some mischance the war does not end in 1918, but lasts into the year 1919. Does that discredit him? Not at all. Consider the case of President Wilson again; in 1919 he will be 63 years old, and will have ruled 7 years, so we have

$$1856+1912+7+63=3838,$$

$$3838\div 2=1919;$$

therefore the war will end in 1919. The argument is all right, and the only trouble was that we applied it prematurely.

Thus the genius of this wonderful seer, like that of Shakespeare, grows more and more impressive as we study his works more deeply. He is worthy of a place in the Hall of Fame beside those scholastic philosophers of the Middle Ages, who settled theological questions with so much ingenuity by the aid of the mystic properties of numbers.

ORIENTATION FOR HEAVY (COAST) ARTILLERY

The methods used by the light artillery, so excellent for rapid work in open warfare, break down when used with large caliber artillery at long range. The errors introduced by knowing only approximately the positions of target and battery and by making an estimation of the effect of wind, atmosphere, and similar disturbing causes are too large to be allowable. It is therefore necessary to make corrections for non-standard conditions and to know accurately the location of the battery and target and their difference in level. There are a number of considerations which require that the errors in the location of points be negligible as compared to those which can not be avoided, as, for example, erroneous assumptions as to initial muzzle velocity or atmospheric conditions. This requires such careful surveying that the work is usually assigned to men especially trained for the purpose, called "Orientation Officers." However, all Coast Artillery Officers are expected to be able to do this work.

Among the duties of an Orientation Officer are the location of possible positions for our batteries, and the determination of the co-ordinates of these positions, and of the targets on which it may be intended to fire, together with observation stations and aiming points. To be able to accomplish all this successfully requires a thorough knowledge of the transit and the plane table, also of the making of maps, of map projections, the accuracy to be obtained with a given projection, drafting and sketching, elementary astronomy, and the gunnery of the batteries being served.

The choice of a system of map projection depends on the requirements of the map. In civil maps, for example, it is desirable to have areas invariant, while an artillery map must give angles very accurately and distances with only a small error.

For the part of the Western Front occupied by the French, and now, to a certain extent, by our forces, the Lambert Projection gives best results. Points on the earth are projected by mathematical formula on to a cone which cuts the earth along parallels 53 grades north and 57 grades north latitude. The origin of this projection is at longitude six grades east of Paris,

latitude fifty-five grades north. A kilometric grid is placed over this map, with the grid lines parallel and perpendicular, respectively, to the meridian of the origin. This origin is taken as the origin of a system of rectangular coordinates. The coordinates of two points being known, the solution of a right triangle gives the length and direction of the line joining them.

The angle between the meridian and the grid line through any point is given by $\alpha = (M+6).76$, where M is the longitude of the point expressed and α is in grades. This longitude is measured from the meridian of Paris and is negative if east, positive if west. Direction, computed from coordinates, is measured from "Grid North" in a clockwise direction, and is called "Y- Azimuth." To find the true Azimuth of this direction the value of α must be computed.

Difference in level between battery and target is usually found from the contour lines of the map. By means of profiles taken directly from the contours a study is made for visible and invisible areas, (defilade) and mask. The dead area for a given battery position is found from profiles radiating from the battery. This is the area which cannot be reached by fire because the ground between it and the battery is steeper than the slope of fall of a projectile fired at the given range. An interesting application of profiles is the determination of the necessary height of a captive balloon, sent up at some point within our lines, in order that it may be able to detect an enemy battery behind a steep hill.

Among the methods used to determine the coordinates of a point are: intersection, resection, and traverse. The coordinates of a given position can be found by running a transit traverse from some known triangulation point, which has been located very accurately. To run this traverse it is necessary to orient the transit. This may be done by sighting between two known stations or by observations on the sun, on polaris, or on a star at equal altitudes. If a star is observed when at the altitude on both sides of its orbit. True North is half way between the two directions. No computations are necessary. Observations on Polaris or the Sun must be reduced and for this

purpose the necessary tables should be included in every artilleryman's note book.

An approximate orientation can be secured with the magnetic compass if the declination is known.

Intersection is often used to determine the coordinates of a target which can be seen from two or more stations within our line.

Resection, which will be explained rather in detail, is used to locate points which we can occupy and from which several known points can be seen.

The plane table may be used if it is not necessary to have the coordinates closer than ten or twenty meters. For more accurate work a transit should be set up and the angles measured at the unknown point between lines to the known points. If the transit is oriented it is necessary to sight on only two known points.

With an instrument which is not oriented three points are required to give a solution. This is the so-called "Three Point Problem." In addition to a number of solutions with the plane table, already mentioned, there are several mathematical solutions, one of which will be given.

Call the unknown point P and the three known points A , C , and B , lettered from left to right. Measure, at P , in a clockwise direction, the angle from PC to PB , and call this angle α .

Likewise measure the angle from PC to PA , in a clockwise direction, and call it β . Transform the coordinates to a new origin at C with the axes parallel to the old axes. Then the new coordinates are: $C(0, 0)$, $A(X_1, Y_1)$, $B(X_2, Y_2)$. Let P be designated by (X, Y) .

The point, P , is at the intersection of two circles: one passing through A and C and capable of containing the angle $A P C$; the other, passing through B and C and capable of containing the angle $B P C$. The problem is indeterminate if these two circles are coincident, which is the case when P is on the circle through A , C , and B . We will proceed to give the solution for x and y , the coordinates of P , for the case, when the problem can be solved.

$$\tan \beta = \frac{\frac{y}{x} - \frac{y-y_1}{x-x_1}}{1 + \frac{y(y-y_1)}{x(x-x_1)}} \quad \text{which reduces to}$$

$$x^2 + y^2 - (x_1 + y_1 \cot \beta)x + (x_1 \cot \beta - y_1)y = 0 \quad \dots (1)$$

Similarly the circle P C B is given by

$$\tan \alpha = \frac{\frac{y}{x} - \frac{y-y_2}{x-x_2}}{1 + \frac{y(y-y_2)}{x(x-x_2)}}$$

which becomes

$$x^2 + y^2 - (x_2 + y_2 \cot \alpha)x + (x_2 \cot \alpha - y_2)y = 0 \quad \dots (2)$$

Subtract (2) from (1) and we have

$$(x_1 \cot \beta - y_1 - x^2 \cot \alpha + y^2)y = (x_1 + y_1 \cot \beta - x_2 - y_2 \cot \alpha)x$$

$$Y = \left(\frac{x_1 - x_2 + y_1 \cot \beta - y_2 \cot \alpha}{y_2 - y_1 + x_1 \cot \beta - x_2 \cot \alpha} \right) X \quad \dots (3)$$

$$\frac{x_1 - x_2 + y_1 \cot \beta - y_2 \cot \alpha}{y_2 - y_1 + x_1 \cot \beta - x_2 \cot \alpha} = K \quad \dots (4)$$

substitute

Kx for y in (1)

$$x^2 + k^2 x^2 - (x_1 + y_1 \cot \beta)x + (x_1 \cot \beta - y_1)Kx = 0$$

$$X = \frac{x_1 + y_1 \cot \beta + (y_1 - x_1 \cot \beta) K}{1 + K^2} \quad \dots (5)$$

Similarly substitute $y = kx$ in (2)

and we have

$$X = \frac{x_2 + y_2 \cot \alpha + (y_2 - x_2 \cot \alpha) K}{1 + K^2} \quad \dots (6)$$

To work a numerical problem, find K from (4), then compute x in parallel columns from (5) and (6), finally $y = kx$.

Care should be taken that the proper signs are given

$$x_1, y_1, x_2, y_2, \cot \alpha \text{ and } \cot \beta.$$

When the known points can not be seen from the desired position a short traverse is run to a point from which they can be seen.

It is necessary for the Orientation Officer to have a very clear understanding of his duty in order that he may train his men to do the work for which he is responsible and that he may be able to plan intelligently and see that his plans are executed.

CAPT. E. J. OGLESBY,

Coast Artillery.

PROGRESSIVE TEACHING OF MATHEMATICS*

BY G. W. MYERS,
UNIVERSITY OF CHICAGO

If I were a sufficiently staunch republican in politics, as I used to be, I would not admire the adjective in my title. I presume I should prefer the phrasing "stand-pat" mathematical teaching. But nowadays I am not sufficiently staunch. If I were sufficiently "dyed-in-the-wool" democratic, perhaps I should prefer "Reform" mathematical teaching. But I am not sufficiently partisan even at this angle of bias. Furthermore, I shall assume that there is amongst you a sufficient degree of political detachment to tolerate the adjective, progressive, as a qualifier of mathematical teaching.

What shall a mathematical teacher do today to merit being regarded as a progressive teacher by his professional colleagues? In this day of school surveyors, gap-bridgers, critics, iconoclasts, and general fault-finders, when to the staid educationists it looks as though the experts were running amuck through the educational world in general, and through mathematical education in particular, it is not easy to answer this question well. The answer, however, is none the less desirable, and I shall at least try to give my form of answer.

THE ATTITUDE OF THE PROFESSIONAL TEACHER

In the first place, what one should do to be regarded as a progressive mathematical teacher must depend on the nature of the situation in which he has contracted to teach. If one must teach under a rigid curriculum, fixed by the authorities beyond the pale of redemption activities, he should neither chafe nor quail, but calmly accept the inevitable. The progressive teacher in such a situation, and this type of situation is numerous, will rather direct his efforts at making the very best he can out of the existing order than waste them in trying to rem-

*School Science and Mathematics, May, 1918.

edy the irremediable. He will do best by following the conventional procedure. But even here he may avail himself of the pedagogic fact that we spend too large a proportion of our time and effort in merely exemplifying and illustrating mathematical principles and too small a proportion in teaching how to solve social and everyday problems with the aid of the mathematical tools. Even in such a situation I would enrich the work with more applications in which the toolage advantages of the school mathematics would become apparent. This much may be attempted by even the timid teacher without jeopardizing his position.

If the fixed curriculum were distinctly poor, I should now and again hint of ways in which it might be made better. I would, however, be careful to let the principal or superintendent have the first chance at the honor of doing the improving, and would undertake it myself only on the request of this usually jealous official. Of course, the main reason that mathematical teachers have influenced mathematical curricula so slightly is that they have been allowed so little opportunity to influence it. But when educational people go about saying that the mathematical specialist has had his day with the curriculum, has failed to profit by it, and that now the improvement of mathematical courses should be turned over to others, it is well to remember that "his day" was usually usurped by some "superior official." In the fixed situation the progressive teacher of mathematics will strive to be "as wise as a serpent and as harmless as a dove."

There are, however, numerous situations in which a level-headed mathematical teacher can do about as nearly as he pleases with the mathematical course of study and the special courses that make it up, as a single member of a necessarily correlated corps of teachers ought to want to do. The kind of use the favored members of our profession who function in such situations make of their opportunities is of the very highest significance for the improvement of our work. It is they to whom we are to look for the first formulations of general plans for new advances and for the smoothing out of the first steps in bringing the new formulations into practicable shape for aver-

age conditions. The progressive teacher whose good fortune has placed him in one of these favored situations will today give close attention to the following points:

I. He will see to it that his attitude toward attacks from professional critics upon his subject and upon the results of his work as teacher is professionally correct. Attitude in mathematical matters means much more than many people think. For example, no clearer basis for differentiating the separate fields of arithmetic and algebra has yet been offered than that of attitude toward problems. If the attitude is to find the solution of this particular problem, the problem is one of arithmetic. If it is to find the solution of this type, or class, of problem, we have an algebraic problem. Right attitude implies the correct position for an economical start. "Well begun is half done," is more than an adage in mathematical matters. But the particular phase of attitude I am here emphasizing is attitude toward attack. It is no secret that we are being vigorously attacked nowadays as to subject-matter and methods.

Of the attitudes that have found spokesmen from one quarter or another the following four are prominently in evidence:

1. The neutral attitude.

This attitude expresses itself in such phrases as, "Mathematics is good enough. Let it alone." If you ask us, "What are we going to do about it," we answer, "Nothing." "What was good enough for grandfather, is good enough for us." Their counsel is: "Go into the cyclone cellar and wait for the storm to pass." In spite of some very disturbing facts against the adequacy of the results and the time and effort of mathematical teaching in public high schools that entirely conscientious and capable experts have brought out to view, the neutral says: "This is no concern of mine," or "Let the heathen rage," or "Those crazy folk, having no real subject-matter to talk about publicly, must trump up talk about some bugaboo that will give them a hearing." The neutral attitude is very far from the attitude that the progressive teacher will take.

2. The belligerent attitude.

The spokesmen of this attitude counsel: "Blow up the enemy." "Get in the trenches of tradition and convention and 'stay

put'." Or in the patois of partizan politics their counsel is "stand pat." They attack the character of the enemy by suspecting that he had early trouble with his school mathematics and can't forget it, or that he now has son or daughter who cannot learn mathematics. But this "mailed fist" type of argument gets nowhere, convinces nobody, and harms only its user. The progressive mathematical teacher will shun it as he would the plague, for it is stultifying to every participant.

3. The reverential attitude.

This is the attitude of reverence for the past. What educators and thinkers for over two thousand years have regarded as fundamental and essential educational material must now be called in question. It is a sort of educational sacrilege to pit one's individual judgment against the consensus of the ages. The past fruits of present curricular forms in the lives of great men or successful men, who testify to its benefits, ought to quiet attacks upon their present worth. This attitude is a flat denial of progress, fixes the stationary upon us, and cannot be assumed by the progressive mathematical teacher of today. It is the ancient Egyptian view and is unprogressive. We dare not forget—

"New occasions teach new duties, time makes ancient good uncouth."

In this day, when men's views are undergoing profound changes, when long cherished ideals are being replaced by new ones almost over night, we cannot hope for appreciable help from the reverential attitude.

4. The psychological attitude.

In the last twenty-five years a new attitude toward educational questions has been gradually working its way into the minds of men. Many middle-aged and older teachers of the conventional school subjects, teachers whose standards of educational worth were worked out to their own satisfaction before the new attitude had matured, have no adequate appreciation of the worth of psychological norms and standards, even though they may have found time now and then in their busy pursuits to do a little desultory reading in psychology. Indeed, it is the failure to recognize the psychological attitude toward educational ques-

tions that furnishes representatives for the three attitudes enumerated above.

The reason for the discomfiture, where it exists, among mathematical specialists, particularly among professors of mathematics in colleges and universities, at the recent propaganda against school mathematics as organized and administered in current curricula, is that these persons have no psychological point of view at all. It is not a wrong point of view that causes the discomfiture, it is the complete lack of a psychological point of view. This is also the cause of the repeated appearance of the same old objectionable form of textbook for high school mathematics. The author has no psychological attitude at all toward the problem of mathematical teaching, which is pre-eminently a psychological problem. The attitude of the author is either empirical—mere habituated classroom procedure—or it is purely mathematical. The author tries to produce a good text. He has no malice toward a good text. He simply doesn't know what a good teaching text is separate and apart from the grooves of his habituated practice. His weakness is not glaringly revealed yet, for the choosers of texts are as lacking in psychological attitude as he is. Moreover, the users of the text thus chosen have been merely habituated to the same type as has the author himself. The whole cause of the attitude of pugnacity toward the school critics of mathematics grows out of lack of any psychological attitude, while at the same time the tenacity of the present unsatisfactory status of high school mathematics is due to the same lack in school people who occupy critical places between the author and the child. The progressive teacher will not unduly quarrel with this status of things. He will simply recognize it as a fact, will see to it that he himself has a psychological attitude, will seek encouragement in the thought that as the more recently qualified recruits to the ranks of teaching become more numerous and influential this condition will be corrected, and he will regard these extra-mural attacks as corrective agencies. The progressive attitude must be the psychological attitude.

The psychological attitude implies an ability and a willingness to see a fault, even when it is one's own. It demands open-

mindfulness to arguments both for and against. It does not regard change, even great change, as sacrilege, but rather as a professional need, if not necessity. It meets argument with argument. It neither coaxes nor cajoles; neither grouches nor grumbles; resorts to neither poisoned gases nor organized resistance. It believes in the ultimate perseverance of the saintly. It is always pacifically perseverant along established lines of principle. The psychological attitude is the progressive attitude today.

THE VIEW OF THE PROFESSIONAL TEACHER

II. Right attitude starts one right, but right views are what hold him right. The progressive teacher must not only face and start right, he must keep right. Here again there are four aspects of educational view that the progressive teacher will not neglect.

1. The educational view.

Of the vast body of mathematical material from which we might select what we will use for educational purposes, some parts are more appropriate than others. The educational view is that the mathematical material needed for the curriculum must be of the sort that faces both inward and outward. For example, the triangle is important because of its general relatedness to all sorts of plane rectilinear figures. The properties of nearly all plane figures may be derived by analyzing the figure into some form of triangle, after the properties of the triangle itself have been made out. This is the inward face of the theory of the triangle and from it comes the scientific value of the triangle. But many problems of construction, of surveying, etc., that arise in everyday life are solved by the aid of the theory of the triangle as a tool for general non-mathematical use. This is the outward face and from it the triangle gets a utilitarian value. Those parts of the general body of mathematical material that have both faces, the outward and the inward, are what should furnish curricula with their material. This principle will be an accepted and working view of the progressive mathematical teacher.

2. The mathematical view.

The mathematical view is that mathematical principles face only inward. Perhaps no high school teacher holds this as a conscious view for the control of teaching, but the difficulty is that he teaches, quite unconsciously, as though this were his view. He uses vocational or physical situations to exemplify, or to illustrate, mathematical principles, and uses them only in this way. He rarely, if ever, uses mathematical technique purely as a tool to resolve some scientific, or social or industrial situation. The text rarely, if ever, furnishes such situations for mathematical exploitation, and it is doubtful whether it should. But the teaching should not omit nor even minimize this instrumental use of mathematics, for most young persons get whatever confidence they have in the worth of mathematical study from the feeling that such study confers power over the quantitative world. They develop an interest in the inward aspect of mathematical relatedness, some slowly, some rapidly, but all gradually as the mathematizing proceeds to later stages. Faith and interest in mathematizing proceeds to later stages. Faith and interest in mathematical effort of both sorts, both that which aims at mathematical insight and grasp and that which exhibits power of use, must be the continual aim of the progressive teacher. He needs to be particularly careful not to read his own mathematical interest into the early classes. He should with them be continually employing the utilitarian possibilities on local and familiar problematic situations of an extra-mathematical character. Much more than the customary emphasis upon real applications in which well mastered mathematical principles function as real tools is what I am claiming the progressive teacher should do today.

3. The social view.

A third view is that school mathematics should have high value as giving social, occupational, and vocational insights and power. We have not yet found practical ways of doing this. But we must not yet give up the task. It means teaching the formula, the question, the function, the graph, logarithms, etc., much earlier than we do. This is the social view, and is one of the dominant notes of secondary education today. The progress-

ive teacher will find some way of getting himself into friendly relations to it in practice. He will at least not ignore it, nor oppose it.

4. The rational view.

A fourth view that the progressive mathematics teacher must employ explicitly is what we shall call the rational view. It is that the teacher should charge himself with the professional duty of training his pupils to take a rational attitude toward their school tasks, to the end that they may come to assume this attitude toward the tasks of life. We know that youth are anxious to do the mere technique of mathematical work, and that they shrink from, sometimes even resent, trying to give the reasons for the doing. By the persistent "whys" of mathematics we can and should break up this shallow satisfaction and establish the real student attitude of wanting to know why. Progressive teachers will train to the rational attitude in mathematical study. This is a widespread and a crying need in our public schools.

The progressive mathematics teacher will keep his mind clear on all four of the views, the educational view of mathematics, the mathematical view, the social view, and the rational view, and conform to all of them as his ability and experience show him profitable ways of doing so.

THE VALUES OF MATHEMATICAL EDUCATION

Right attitude and right views prepare the professional teacher for a scrutiny of the values of mathematical education. Critics of our work have claimed that there is no general agreement amongst us as to the aims and values of our work, and that whatever they are, our work is not realizing them with large numbers of our pupils. This, they say, is a strong reason why our subjects should be made elective. It is not my purpose here to enter a rejoinder. I do say that we may regard the following values of good school work in mathematics as almost, if not quite, universally accepted, and that it is the duty of the professional teacher to accept them and to undertake explicitly to secure an appreciable measure of all of them with all his pupils.

1. The formal value.

This means the value represented by a facile measure of skill in the direct use and manipulation of mathematical symbolism. Mechanical though it be, one will hardly make any sort of use, instrumental or scientific, of mathematics, if a considerable part of his mental energy is used up in the mechanics of mere symbol manipulation, and only the residuum is available for energizing against the problem. Without sufficient skill in mathematical mechanics for easy manipulation, mathematical symbolism actually encumbers thought, and is a nuisance. Enough skill to keep mechanics out of the way of thought is the ideal, and the progressive teacher will not fail to strive at least for this degree of formal skill.

2. The literary value.

This means the value represented by the ability to interpret popular scientific articles and books. This ability is needed by every high school pupil, and the progressive teacher of mathematics will secure it.

3. The instrumental value.

This is only a higher measure, which can be attained by many high school pupils, of the literary value. It means the ability to read and interpret intelligently scientific and professional subject-matter of the precise quantitative sort. It is to include also the ability to use the technique of mathematics in dealing with scientific and engineering facts and principles. The progressive teacher will give considerable attention to attempts to realize this value, especially in the later courses.

4. The scientific value.

This value is intended to include what may be called the vocational or mathematical value. It is such a degree and kind of mastery of elementary mathematical *technique and rationale* as the prospective specialist in mathematics must have. This value perhaps need not be insisted on here, for it is the sort of value we are now almost universally attempting to secure for all. I mention it here because it must not be neglected by the progressive teacher, even though other values are just now crying loudly for more attention than we are giving them.

5. The transfer value.

Mathematicians and mathematical teachers are in sufficiently full agreement as to whether this phase of value exists, and the professors of education, formerly considerably at loggerheads about it, are coming to an agreement that something carries over. Precisely what it is that carries over, and to what extent it carries over, remains yet to be answered. That some sort of gain in attitude toward problem thinking and some measure of the technique of problem solving goes over into fields in which problem thinking is characteristic is conceded by nearly everyone. If we should make our teaching such as to call for a great deal more problem thinking from the pupil, we shall surely be increasing this value of mathematics. Learning model demonstrations is not problem thinking, nor is symbol juggling. Problem thinking is analyzing, organizing and controlling problematical situations—situations which produce the dilemma attitude in the learner's mind—and we do too little with this sort of thing in our classes. There is too much "fixing things up" so that the pupil may never be troubled by a dilemma, too much teaching him everything he needs before he needs it, to admit of much training in problem thinking in our present day teaching. The progressive teacher will produce some dilemmas for the pupil himself to dispel. More important mathematical things should be taught obliquely, as tools needed in problem solving.

NORMS FOR GAUGING MATHEMATICAL VALUES

The progressive teacher will seek to aid in setting up standards for judging the special achievements of mathematical study. Such questions as: "Does an ever widening deviation of abilities in your class indicate good, bad, or indifferent teaching?" can be answered by the high school teacher in the pursuit of his regular work. I raise the question: "Why is an ever widening deviation abilities in high school classes in mathematics so general today?" The progressive teacher will associate himself as occasion offers, as our chairman has been long doing, with some professor of education and help to set up suitable study tests, ability tests, and attainment tests. While the psychological and the mathematical attitudes are so divergent as

they are, this is a practicable way of getting both attitudes in function. Perhaps a standard grading system might be worked out thus. We always have a higher regard for the ideas of a man with whom we have worked than we have from just hearing him recite our weakness. Let us get together. Instead of chafing under the findings of the school surveyors, it will be much more progressive for us to help set up more adequate standards, and then after weaknesses have revealed themselves let us be broad-minded enough to bestir ourselves to remove them.

A CLOSING REMINDER

Just one more reminder in closing. Nearly every educational writer and general educational authority who has recently tried to give us suggestions for constructive improvement has urged that we introduce the formula, the graph, and logarithms in the first year high school course in mathematics. None of us is even attempting to do this, nor has any good reason for not trying this as yet appeared. Why is this? Do actual teachers regard these suggestions for modernizing high school algebra as of no value? Has anyone tried out these suggestions? The recent texts are doing a little half, or quarter-hearted work with the graph. But the formula and logarithms are not seriously tried by anyone. Why is this? I close with the question, without even asking why not introduce the function idea earlier in algebra.

THE STRAIGHT EDGE

Let every mathematics teacher who wants to learn something or who thinks he has something to impart make a "straight line" for Dallas Thanksgiving.

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Nobody has started the slogan, "Mathematics will win the War," but just the same it is making notable contributions to that end.

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Are you going to be able to tell your students something of the part mathematics has had in the war? If not you ought to get a search warrant and go hunting for some information.

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What have you read on the subject of mathematics or its teaching since school closed last spring?

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Can you honestly say to yourself that you think you are better equipped for your work than you were last year?

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Come to Dallas Thanksgiving. Some of the best teachers in Texas are going to be there to tell us a lot of things about the very points that are troubling us.

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Come to Dallas. If you think you are not up to the crowd, just come, sit tight, be quiet, look interested, and it will be imputed to you for wisdom.

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How many mathematics teachers in Texas do you know personally? Come and make the acquaintance of your fellows. They are worth knowing or they would not be teaching the great subject that they are.

